

FIND MISSING SIDES OF SIMILAR TRIANGLES Study Guide

Find Missing Sides of Similar Triangles: A Comprehensive Guide

Find missing sides of similar triangles is a fundamental skill in geometry that helps students and professionals alike solve complex problems involving shapes and measurements. Similar triangles are triangles that have the same shape but not necessarily the same size. They are proportional, meaning their corresponding sides are in proportion. Mastering how to find missing sides allows for accurate calculations in various fields such as engineering, architecture, and everyday problem-solving.

Understanding Similar Triangles

What Are Similar Triangles?

- Two triangles are similar if their corresponding angles are equal.
- Corresponding sides are in proportion.
- The notation for similar triangles is often expressed as $\triangle ABC \sim \triangle DEF$, indicating triangle ABC is similar to triangle DEF.

Properties of Similar Triangles

- Corresponding angles are equal.
- Corresponding sides are proportional.
- The ratios of corresponding sides are equal: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

Why Is It Important to Find Missing Sides?

- To determine unknown measurements in geometric figures.
- To solve real-world problems involving distances and heights.
- To prove the similarity of triangles and other geometric properties.

Methods to Find Missing Sides of Similar Triangles

Using Proportions

The primary method involves setting up proportions based on the similarity of triangles.

Step-by-step process:

- Identify the known sides and the corresponding sides that are missing.
- Write down the proportion equations comparing the known and unknown sides.
- Cross-multiply to solve for the missing side.

Example:

Suppose you have two similar triangles, with sides:

- Triangle 1: sides 8 cm, 15 cm, 17 cm.
- Triangle 2: sides x, 30 cm, 34 cm.

To find the missing side x:

- Match sides proportionally: $\frac{8}{x} = \frac{15}{30} = \frac{17}{34}$.

Calculate x using the ratio:

$\frac{8}{x} = \frac{15}{30}$

$$\frac{8}{x} = \frac{15}{30} \rightarrow 8 \times 30 = 15 \times x \rightarrow 240 = 15x \rightarrow x = \frac{240}{15} = 16, \text{ cm}.$$

$\frac{17}{34} = \frac{15}{30}$

Similarly, for the side corresponding to 17 cm:

$\frac{17}{34} = \frac{15}{30}$

$$\frac{17}{34} = \frac{15}{30} \rightarrow 17 \times 30 = 15 \times 34 \rightarrow 510 = 510,$$

$\frac{17}{34} = \frac{15}{30}$

which confirms the proportionality.

Using Scale Factors

- Determine the scale factor between the two triangles.
- Use the scale factor to find missing sides.

Steps:

- Find the scale factor (k) by dividing a known side in the larger triangle by its corresponding side in the smaller triangle.
- Multiply any known side in the smaller triangle by (k) to find the missing side.

Example:

If the larger triangle has a side of 20 units and the smaller triangle has a corresponding side of 10 units:

- Scale factor $(k = \frac{20}{10} = 2)$.

To find a missing side in the smaller triangle corresponding to a side of 14 units in the larger triangle:

- Smaller triangle side $= (\frac{14}{k} = \frac{14}{2} = 7)$ units.

Using Geometric Ratios and Properties

- Use ratios of sides based on given angles or other known measurements.
- Apply Pythagoras theorem if the triangles are right-angled.

Special Cases and Tips

Right-Angled Similar Triangles

- Use Pythagoras' theorem to find missing sides if the triangles are right-angled.
- Remember that in right triangles, the hypotenuse and legs are related via $(a^2 + b^2 = c^2)$.

Common Mistakes to Avoid

- Mixing up corresponding sides.
- Incorrectly assuming sides are proportional without verifying similarity.
- Forgetting to reduce ratios to simplest form before solving.

Practice Problems and Solutions

Problem 1

Two similar triangles have sides 6 cm, 8 cm, and 10 cm for the smaller triangle. The larger triangle has one side measuring 15 cm, which corresponds to the 8 cm side in the smaller triangle. Find the other two sides of the larger triangle.

Solution:

- Find the scale factor:

\[

$$k = \frac{15}{8} = 1.875.$$

\]

- Find the other sides:

\[

$$\text{Side corresponding to 6 cm} = 6 \times 1.875 = 11.25, \text{ cm}.$$

\]

\[

$$\text{Side corresponding to 10 cm} = 10 \times 1.875 = 18.75, \text{ cm}.$$

\]

Answer: The other sides are 11.25 cm and 18.75 cm.

Problem 2

In two similar right triangles, one has legs of 9 cm and 12 cm, and the hypotenuse of the large triangle is 15 cm. Find the hypotenuse of the smaller triangle.

Solution:

- Find the hypotenuse of the smaller triangle using Pythagoras:

\[

$$\text{Hypotenuse} = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15, \text{ cm}.$$

\]

- Since the triangles are similar and their hypotenuses are both 15 cm, the scale factor is 1, implying the smaller triangle's hypotenuse is 15 cm, matching the larger.

Note: In this case, the triangles are congruent in shape, and their sides are proportional with a scale factor of 1.

Conclusion

Finding missing sides of similar triangles is a vital skill that combines understanding of proportionality, geometric properties, and algebraic manipulation. Whether working with classic triangle similarity or applying Pythagoras' theorem in right triangles, the key lies in correctly identifying corresponding sides, setting up accurate proportions, and carefully solving for unknowns. With practice, interpreting and solving these problems becomes intuitive, making it easier to approach complex geometrical challenges confidently and accurately.

Find Missing Sides of Similar Triangles is a fundamental concept in geometry that allows students and professionals alike to solve complex problems involving triangles efficiently. Understanding how to identify and calculate missing sides using the properties of similar triangles is essential for various applications, from academic examinations to real-world engineering and architectural projects. This article explores the concept thoroughly, providing detailed explanations, step-by-step methods, and practical tips to master this geometric skill.

Understanding Similar Triangles

Before diving into methods for finding missing sides, it's crucial to grasp what similar triangles are and why their properties are so powerful.

Definition of Similar Triangles

Similar triangles are triangles that have the same shape but not necessarily the same size. They are denoted as $\triangle ABC \sim \triangle DEF$, indicating that corresponding angles are equal, and corresponding sides are proportional.

Properties of Similar Triangles

- Corresponding angles are equal ($\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$).
- Corresponding sides are proportional: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

Significance in Finding Missing Sides

These properties form the basis for calculating unknown lengths. Once the ratios of known sides are established, you can apply proportional reasoning to determine missing sides, even when direct measurements are unavailable.

Methods to Find Missing Sides

There are several techniques to find missing sides of similar triangles, primarily involving ratios, proportions, and sometimes trigonometry. Here, we explore the most common and effective methods.

Using Proportionality of Corresponding Sides

This is the most straightforward method, applicable when at least two pairs of corresponding sides are known.

Step-by-step Approach:

- Identify the corresponding sides of the similar triangles.
- Write the proportion equations: $\frac{\text{Known side in triangle 1}}{\text{Known side in triangle 2}} = \frac{\text{Unknown side in triangle 1}}{\text{Corresponding side in triangle 2}}$.
- Cross-multiply and solve for the missing side.

Example:

Suppose two triangles are similar, with sides:

- Triangle 1: AB = 8 cm, BC = 12 cm, AC = 15 cm.
- Triangle 2: DE = 4 cm, EF = x, DF = 7.5 cm.

Given that AB corresponds to DE, BC to EF, and AC to DF, find EF.

Proportion:

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{8}{4} = \frac{12}{x}$$

Cross-multiplied:

$$8x = 4 \times 12$$

$$8x = 48$$

$$x = 6, \text{ cm}$$

Advantages:

- Simple and direct.
- Requires only known side lengths.

Limitations:

- Only applicable when at least two sides are known.

Using Ratios and Scale Factors

When the scale factor between two similar triangles is known, missing sides can be found by multiplying known sides by this factor.

Steps:

- Determine the scale factor (k) by dividing a known side in the larger triangle by its corresponding side in the smaller triangle.
- Multiply the known side of the smaller triangle by (k) to find the corresponding side in the larger triangle, or vice versa.

Example:

Given:

- Triangle A: sides 6 cm, 8 cm, 10 cm.
- Triangle B: similar to A, with one side known as 12 cm.

Find the scale factor:

$$(k = \frac{12}{6} = 2).$$

Find other sides:

- For the side corresponding to 8 cm: $(8 \times 2 = 16, \text{ cm})$.
- For the side corresponding to 10 cm: $(10 \times 2 = 20, \text{ cm})$.

Advantages:

- Efficient when scale factor is known.
- Useful for complex figures with multiple similar triangles.

Limitations:

- Requires at least one side to be known in the larger or smaller triangle to find the scale factor.

Using Basic Geometry and Trigonometry

In scenarios where angles are known but sides are missing, trigonometry becomes essential.

Common techniques include:

- Law of Sines
- Law of Cosines

These can be used to find missing sides when angles are known, and the triangles are similar but not necessarily right-angled.

Example:

Suppose in similar triangles, you know two angles and one side. Using the Law of Sines:

\[

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

\]

You can find missing sides.

Practical Applications and Tips

The ability to find missing sides of similar triangles is invaluable in many fields. Here are some practical applications and tips:

Applications

- Architecture & Engineering: Scaling models and structures accurately.
- Navigation & Surveying: Calculating distances indirectly.
- Art & Design: Maintaining proportions in scaled drawings.
- Physics: Modeling similar systems where direct measurement is impossible.

Tips for Success

- Always verify the correspondence of sides and angles before applying ratios.
- Use diagrams to visualize the triangles and label all known and unknown sides clearly.
- When multiple methods are possible, choose the most straightforward one to minimize errors.
- For complex problems, combine methods?use ratios, proportions, and trigonometry as needed.

Common Challenges and Solutions

Challenge 1: Identifying which sides correspond.

- Solution: Use angles, labels, and diagrams carefully; confirm that sides are opposite equal angles.

Challenge 2: Insufficient data.

- Solution: Look for auxiliary information or angles that can help apply trigonometry or additional similarity ratios.

Challenge 3: Calculation errors.

- Solution: Cross-check calculations and ensure ratios are correctly established.

Summary of Key Points

- Similar triangles have proportional sides and equal angles.
- Finding missing sides often involves setting up proportions based on known sides.
- The scale factor is a powerful tool when available.
- Trigonometry provides solutions when angles are known, and sides are missing.
- Accurate diagramming and labeling are crucial for problem-solving.

Conclusion

Mastering the skill of finding missing sides of similar triangles enhances geometric problem-solving capabilities and underpins a wide range of practical applications. Whether through simple ratios, scale factors, or advanced trigonometry, understanding the underlying principles ensures accuracy and efficiency. With practice, identifying and calculating unknown sides becomes intuitive, enabling students and professionals to approach complex geometric challenges with confidence.

Question How can I find the missing side of a similar triangle if I know two sides of one triangle and two sides of the other? Use the property of similar triangles that corresponding sides are proportional. Set up a proportion between the known sides and solve for the missing side accordingly. What is the role of scale factors in finding missing sides of similar triangles? The scale factor is the ratio of corresponding sides of two similar triangles. Knowing this ratio allows you to find missing sides by multiplying or dividing known sides by the scale factor. Can I find a missing side of a similar triangle if I only know one pair of corresponding sides? No, you need at least two pairs of

corresponding sides to set up a proportion and accurately find the missing side, unless additional information is provided. What are common methods to find missing sides in similar triangles besides proportions? Common methods include using the Pythagorean theorem for right triangles or applying properties of special triangles like 30-60-90 or 45-45-90 triangles, in addition to proportions. How does similarity help in solving real-world problems involving missing triangle sides? Similarity allows us to set up proportions based on known dimensions, enabling us to find unknown measurements in real-world contexts such as architecture, engineering, and navigation where direct measurement is difficult.

Related keywords: similar triangles, missing side length, side ratio, proportional sides, triangle similarity, side calculation, scale factor, side length formula, geometric similarity, triangle proportionality

exploring similar figures

Exploring Similar Figures is an essential aspect of mathematical learning and problem-solving. Whether you're a student, educator, or enthusiast, understanding how to identify and analyze similar figures can deepen your comprehension of geometric concepts and improve your ability to solve complex problems. This article delves into the concept of similar figures, their properties, methods for identifying them, and their applications in real-world contexts. By exploring these ideas thoroughly, you can develop a robust understanding of how similar figures function within the broader realm of geometry.

Understanding Similar Figures

Similar figures are geometric shapes that share the same shape but not necessarily the same size. Their corresponding angles are equal, and their corresponding sides are proportional. This proportionality is what defines the similarity between figures, allowing us to compare and analyze shapes across different scales.

Definition and Key Properties

- **Corresponding angles are equal:** If two figures are similar, each angle in one figure corresponds to an angle in the other figure that has the same measure.
- **Corresponding sides are proportional:** The ratios of the lengths of corresponding sides are equal across the figures.
- **Scale factor:** The ratio of any pair of corresponding sides is known as the scale factor, which

determines how much one figure is scaled relative to the other.

Examples of Similar Figures

Common examples include:

- Two triangles with the same angles but different side lengths.
- Circles with different radii (they are similar as they have the same shape, just scaled differently).
- Rectangles with identical proportions but different sizes.

Methods to Determine Similarity

Identifying whether two figures are similar involves verifying specific criteria. Various methods and theorems help in establishing similarity.

AA Criterion (Angle-Angle)

The most straightforward approach for triangles involves the AA criterion, which states:

- If two triangles have two pairs of corresponding angles equal, then the triangles are similar.

This method leverages the fact that if two angles are known to be equal, the third is automatically equal because the sum of angles in a triangle is 180 degrees.

SAS Criterion (Side-Angle-Side)

- If two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal, then the triangles are similar.

SSS Criterion (Side-Side-Side)

- If all three pairs of corresponding sides are proportional, then the two triangles are similar.

Applying Similar Figures in Problem Solving

Understanding how to apply the properties of similar figures is crucial for solving geometric problems, especially those involving unknown lengths or angles.

Finding Unknown Lengths

Suppose you have two similar triangles, and you know some side lengths in one of them. You can find missing lengths in the other by setting up proportions based on the scale factor.

- Identify the known sides and the corresponding sides in the similar figure.
- Calculate the scale factor by dividing known side lengths.
- Use the scale factor to find the unknown lengths by multiplying corresponding sides.

Proving Figures are Similar

To prove two figures are similar, follow these steps:

- Identify pairs of angles or sides that can be compared.
- Use the appropriate criteria (AA, SAS, SSS) to establish similarity.
- Confirm that the proportionality of sides and equality of angles hold true.

Real-World Applications of Similar Figures

Similar figures are not just theoretical concepts; they have practical applications across various fields.

Architecture and Engineering

- Scaling architectural models to real structures involves using similar figures to maintain proportions.
- Designing components that must fit together precisely often relies on proportional reasoning.

Cartography and Geography

- Maps are scaled-down or scaled-up versions of geographic areas. The concept of similar figures helps in creating accurate representations.

Art and Design

- Artists use proportions and similar figures to create realistic sketches and sculptures.

Advanced Topics in Similar Figures

As you deepen your understanding, exploring advanced concepts related to similar figures can enhance your problem-solving toolkit.

Similarity Transformations

These are geometric transformations that produce similar figures through scaling, translation, and rotation. The key idea is that the shape remains unchanged, only the size may vary.

Coordinate Geometry Approach

- Using coordinate geometry, you can verify similarity by calculating side lengths and angles based on coordinate points.
- Transformations can be algebraically represented to demonstrate similarity.

Congruence vs. Similarity

- While similar figures share shape but not size, congruent figures are identical in both shape and size.
- Understanding the distinction helps clarify when figures are exactly the same or just proportionally related.

Conclusion

Exploring similar figures is fundamental in understanding the relationships between geometric shapes. By mastering the properties, criteria for similarity, and methods to verify similarity, you can approach a wide array of mathematical problems with confidence. Whether it's calculating unknown lengths, proving figures similar, or applying these concepts in real-world contexts like architecture or cartography, the principles of similar figures are invaluable. As you continue to explore this fascinating area of geometry, remember that the key lies in recognizing proportional relationships and understanding how shapes relate to each other through scaling and transformation. Developing a solid grasp of these concepts will enhance your spatial reasoning and problem-solving abilities, paving the way for further mathematical exploration and practical application.

Exploring Similar Figures: Unlocking Patterns and Connections in Mathematics

In the world of mathematics, figures and shapes are not just static entities; they are gateways to understanding symmetry, proportion, and the underlying structure of the universe. One of the

fascinating concepts that bridges geometry and algebra is the idea of similar figures. Exploring similar figures allows mathematicians, students, and enthusiasts to recognize patterns, make predictions, and simplify complex problems. This article delves into the concept of similar figures, their properties, methods to identify them, and their significance across various fields.

Understanding Similar Figures: The Foundations

What Are Similar Figures?

Similar figures are shapes that have the same form but may differ in size. More precisely, they are figures where corresponding angles are equal, and the lengths of corresponding sides are proportional.

Key Characteristics of Similar Figures:

- Corresponding angles are equal.
- The ratios of the lengths of corresponding sides are equal (called the scale factor).
- The shapes are congruent in form but not necessarily in size.

For example, two triangles that have all three angles equal and the sides proportional are similar. Think of a small triangle and a larger, scaled-up version; they are similar because their angles are identical, and their sides are in proportion.

Why Are Similar Figures Important?

Understanding similarity is crucial because it allows us to:

- Simplify complex problems: By reducing large figures to smaller, similar ones.
- Apply proportional reasoning: To solve for unknown lengths or angles.
- Model real-world phenomena: Such as in architecture, engineering, and natural patterns.
- Recognize patterns and symmetries: Fundamental in fields like crystallography and art.

Properties of Similar Figures: Key Principles

The Angle-Angle (AA) Criterion

One of the most straightforward methods to establish similarity between triangles is the AA criterion. If two triangles have two pairs of corresponding angles equal, then the triangles are similar.

Implication:

Once two angles are known to be equal, the third must also be equal because the sum of angles in a triangle is always 180° . Therefore, the entire shape is similar.

The Side-Angle-Side (SAS) Criterion

If one angle of a triangle is equal to the corresponding angle in another triangle, and the sides including these angles are in proportion, then the triangles are similar.

Application:

Useful when you know a pair of angles and the ratios of the adjacent sides.

The Side-Side-Side (SSS) Criterion

When all three pairs of corresponding sides are in proportion, the triangles are similar regardless of their angles.

Note:

This is a powerful criterion because it doesn't require angle measurements?just side lengths.

Methods to Identify and Construct Similar Figures

- Using Proportionality

To determine if two figures are similar, compare their corresponding sides. If the ratios are equal, the figures are similar.

Steps:

- Identify corresponding sides.
- Measure or note their lengths.
- Calculate the ratios of corresponding sides.
- Confirm if these ratios are equal.

Example:

Suppose triangle ABC and triangle DEF have sides $AB = 6$ cm, $AC = 8$ cm, $BC = 10$ cm; and sides $DE = 3$ cm, $DF = 4$ cm, $EF = 5$ cm.

Ratios:

- $AB/DE = 6/3 = 2$
- $AC/DF = 8/4 = 2$
- $BC/EF = 10/5 = 2$

Since all ratios are equal, the triangles are similar with a scale factor of 2.

- Using Geometric Constructions

Constructing similar figures is a practical way to understand their properties. For example, to construct a triangle similar to a given triangle with a specific scale factor:

- Draw the original triangle.
- Choose a scale factor.
- Use a ruler to multiply sides by the scale factor.
- Draw new points accordingly, maintaining the same angles.

This approach reinforces the concept that similar figures are scaled versions of each other.

- Applying Transformation Techniques

Transformations such as dilation (scaling) preserve shape but change size. When a figure undergoes dilation with a specific scale factor centered at a point, the resulting figure is similar to the original.

Transformation Steps:

- Select a center point.
- Choose a scale factor.
- Draw lines from the center to each vertex.
- Mark new points at scaled distances along these lines.
- Connect new points to form the similar figure.

Real-World Applications of Similar Figures

Architecture and Engineering

Architects often use similar figures to create scaled models of buildings and structures. For example, a small-scale model of a bridge uses similar figures to ensure proportions are accurate, facilitating safe and efficient construction.

Cartography and Map-Making

Maps are scaled-down, similar representations of geographical regions. Recognizing similar figures allows cartographers to translate real-world distances into workable map scales.

Natural Patterns and Biological Structures

Many natural phenomena exhibit similar patterns. The branching of trees, the structure of crystals, and even the arrangement of sunflower seeds display similarity, revealing underlying mathematical principles.

Art and Design

Artists and designers utilize similarity to create perspective, symmetry, and proportional harmony in their works, enhancing aesthetic appeal.

Challenges and Limitations

While the concept of similar figures is straightforward, some challenges persist:

- Identifying Corresponding Parts: In complex figures, determining which sides or angles correspond can be intricate.
- Measurement Accuracy: Precise measurements are essential, especially in practical applications like engineering.
- Non-Uniform Scaling: Some transformations may distort figures, making them non-similar if not purely scaled.

Understanding these limitations helps in applying the concept accurately.

Beyond Triangles: Similar Figures in Other Geometric Shapes

Although much of the focus is on triangles due to their simplicity, the concept of similarity extends to

other shapes:

- Quadrilaterals: Rectangles, squares, and parallelograms can be similar if their corresponding angles are equal and sides are proportional.
- Circles: All circles are similar because they can be scaled to match each other; they share the same shape regardless of size.
- Polygons: Regular polygons of the same number of sides are similar if their corresponding sides are proportional and angles are equal.

Significance in Advanced Mathematics and Science

The exploration of similar figures extends beyond basic geometry:

- Trigonometry: Similar triangles underpin the sine, cosine, and tangent ratios, vital for calculating distances and angles.
- Calculus: Similar figures are used in limits and approximations, especially in concepts like infinitesimals.
- Physics: Similarity principles are fundamental in dimensional analysis, helping scientists derive relationships between physical quantities.

Conclusion: Embracing Similarity as a Mathematical Lens

Exploring similar figures unlocks a powerful perspective in understanding the world around us. From the elegant symmetry of natural objects to the precise engineering of structures, similarity serves as a bridge connecting intuition with formal mathematical reasoning. Recognizing the properties and methods to identify similar figures enhances problem-solving skills and fosters a deeper appreciation for the inherent patterns in nature and human creations.

Whether in classrooms, laboratories, or everyday life, the study of similar figures exemplifies how repeating patterns and proportional relationships shape our understanding of space, form, and function. As we continue to explore the vast universe of shapes, the concept of similarity remains a fundamental tool illuminating the interconnectedness of all geometric forms.

Question What are similar figures in geometry? Similar figures are shapes that have the same shape but may differ in size. They have corresponding angles equal and corresponding sides in proportion. How can you determine if two figures are similar? Two figures are similar if their corresponding angles are equal and their corresponding sides are proportional, meaning the ratios of their lengths are the same. What is the importance of the AA (Angle-Angle) similarity criterion? The AA criterion states that if two triangles have two pairs of equal corresponding angles, then the

triangles are similar. It is a quick way to establish similarity without comparing all sides. Can two figures be similar if their sizes are different? Yes, two figures can be similar even if their sizes differ, as long as their corresponding angles are equal and their sides are proportional. What is the scale factor in similar figures? The scale factor is the ratio of the lengths of corresponding sides in two similar figures. It indicates how much one figure is scaled to get the other. How do you find missing side lengths in similar figures? Use the proportionality of corresponding sides. Set up a ratio of known sides and solve for the unknown, ensuring the ratios are equal. What role do similar figures play in real-world applications? Similar figures are used in map reading, engineering, architecture, and art to create scaled models, analyze proportions, and ensure accurate representations. Are all rectangles similar? Why or why not? Not all rectangles are similar. They are only similar if their corresponding sides are proportional and their angles are equal, which is always true for rectangles, but they must have the same aspect ratio to be similar. What is the difference between similar and congruent figures? Congruent figures are identical in shape and size, while similar figures have the same shape but may differ in size, with their sides proportional. Can two circles be similar? Why or why not? Yes, all circles are similar because they have the same shape, and their sizes can differ. Similarity in circles is based on their radii proportionality; all circles are similar regardless of size.

Related keywords: geometric shapes, shape comparison, figure analysis, shape recognition, shape similarity, geometric figures, shape properties, pattern matching, figure classification, shape identification

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